SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

Borgs
Cheng
Evans
Ward
McQ

2007
YEAR 11

ASSESSMENT TASK #1

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Each section is to be returned in a separate bundle.
- All necessary working should be shown in every question.

Total Marks – 67

- Attempt questions ~~1–4~~ 1–5
- Not all questions are of equal value.
- Full marks may not be awarded for careless or badly arranged work.

Examiner: *F.Nesbitt*

QUESTION 1 (12 marks)

(a) Factorise fully: $4x^2 - 36$ 1

(b) Evaluate: $\log_5 10$ correct to 2 decimal places. 1

(c) (i) Express $0.\dot{4}\dot{3}$ as a geometric series, stating the first term and common ratio. 1

(ii) Use the limiting sum of the series to express $0.\dot{4}\dot{3}$ as a fraction. 2

(d) Solve $4^x - 9(2^x) + 8 = 0$ 3

(e) If $f(x) = x^3 - 3x^2 - 24x - 36$, find $f'(-1)$ 2

(f) Sketch $y = e^x$, marking any intercept(s). 1

(g) Find $\lim\limits_{x \to \infty} \dfrac{2x^2 - 4x}{x^2 - 4}$ 1

QUESTION 2 (14 marks)

(a) Differentiate (without simplifying):

(i) $3x\sqrt{x}$ 1

(ii) $\dfrac{3x}{x^3 - 1}$ 2

(iii) $\dfrac{1}{(3x-2)^4}$ 2

(b) If the roots of the quadratic $x^2 - 8x + 5 = 0$ are α and β , find:

(i) $\alpha + \beta$ and $\alpha\beta$ 1

(ii) $\alpha^2 + \beta^2$ 2

(iii) $\alpha^3 + \beta^3$ 2

(c) If $A(x-1)^2 + Bx + C \equiv x^2$, find A, B and C 2

(d) (i) Graph $y = 4 - x^2$, showing x and y intercepts 1

(ii) Hence or otherwise solve $4 - x^2 \geq 0$ 1

QUESTION 3 (13 Marks)

(a) Given the equation $12y = x^2 - 4x - 32$

(i) Write the equation in the form: $(x-h)^2 = 4a(y-k)$. 2

(ii) Find the coordinates of the focus and vertex. 2

(iii) Find the equation of the axis of symmetry. 1

(iv) Find the x intercepts. 2

(v) Sketch the parabola. 1

(b) The sum of the first two terms of a geometric series is 15 and the third term is 20.

Find the first term and the common ratio if the series does not have a limiting sum. 3

(c) If the events A and B are mutually exclusive and P(A) = 0.3 and P(B) = 0.5

Find:

(i) $P(A \cup B)$

(ii) $P(A \cap B)$ 2

QUESTION 4 (13 Marks)

Solve for x:

(a) $\log_2(x+4) - \log_2(x-2) = 1$ 2

(b) $4^x = 5^{2x-1}$ correct to 2 decimal places. 3

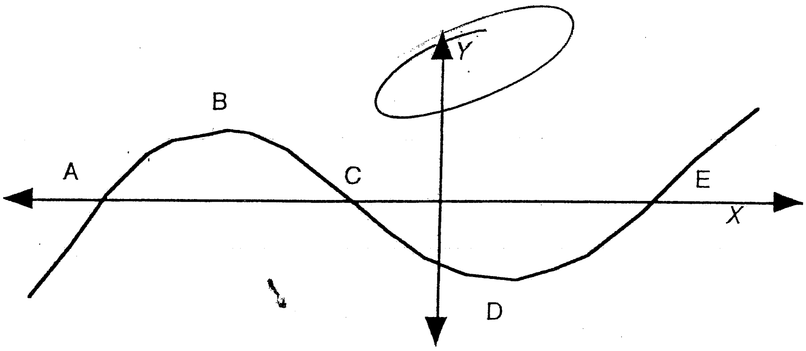
(c) $y = f(x)$ is the equation of a continuous function which passes through the point (0,3).

Find its stationary points and determine their nature if $f'(x) = (x+1)(x-3)$ 3

(d) The tangent and normal to the curve $y = x^3 - x^2 - 6$ at the point P(2,-2) cut the x axis at the points T and N respectively. Find the distance TN. 5

QUESTION 5 (15 Marks)

(a) Below is a sketch of the gradient function of f(x)



(i) Copy the sketch into your booklet.

What can be said about the behaviour of the curve $y =$ f(x) at the points:

(ii) A, C and E? 1

(iii) B and D? 1

(iv) Sketch a possible curve $y =$ f(x), on the same set of axes as (i). 2

(b) \$60 000 is borrowed at 9% p.a. reducible monthly. Fixed repayments are made at the end of each month for ten years.

(i) Using M for each monthly repayment, show that the amount owing at the end of the first month, $A_1 = 60000 \times 1.075 - M$. 1

(ii) Write an expression for A_2, the amount owing at the end of the second month . 1

(iii) Find an expression for A_n and, using the sum of terms of an arthmetic series or otherwise find the value of M. 3

(c) Ship A was 60 km due East of Ship B. Ship A sails at 20 km/h due West and ship B sails due South at 30 km/h.

(a) After 30 minutes how far apart are the ships? (nearest km) 1

(b) Write an expression for the distance between the ships after x hours. 2

(iii) Find the value of x when the ships are closest together. 3

Question 1

(a) $4x^2 - 36 = 4(x^2 - 9)$
$= 4(x-3)(x+3)$

(b) $\log_5 10 = \frac{\log_{10} 10}{\log_{10} 5}$
$= \frac{1}{0.6989}$
$= 1.43$

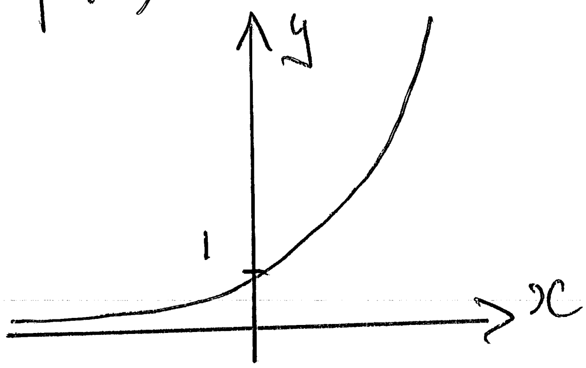
(c) (i) $0.\dot{4}\dot{3} = 0.43 + 0.0043 + 0.000043 + \cdots$
$a = 0.43$ and $r = 0.01$

(ii) $S_\infty = \frac{a}{1-r}$
$= \frac{0.43}{0.99}$
$= \frac{43}{99}$

(d) $4^x - 9 \cdot 2^x + 8 = 0$ → Let $a = 2^x$ then we have:
$a^2 - 9a + 8 = 0$
$(a-8)(a-1) = 0$
$\therefore a = 8, 1 \Rightarrow 2^x = 8, 1$
ie $x = 3, 0$.

(e) $f(x) = x^3 - 3x^2 - 24x - 36$
$f'(x) = 3x^2 - 6x - 24$
$\therefore f'(-1) = 3 + 6 - 24 = -15$

(f)



(g) $\lim_{x\to\infty} \frac{2x^2 - 4x}{x^2 - 4} = \lim_{x\to\infty} \frac{2 - \frac{4x}{x^2}}{1 - \frac{4}{x^2}}$ and $\left.\begin{matrix}\frac{4x}{x^2}\\ \frac{4}{x^2}\end{matrix}\right\} \to 0$ as $x \to \infty$

$= 2$

Question (2)

(a) (i) $3x\sqrt{x}$

$= 3x^{3/2}$

$\frac{d}{dx}(3x\sqrt{x})$

$= \frac{9}{2}x^{\frac{1}{2}}$ 1

(ii) $\frac{d}{dx}\left(\frac{3x}{x^3-1}\right)$

$= \frac{3(x^3-1) - 3x(3x^2)}{(x^3-1)^2}$

$= \frac{3x^3 - 3 - 9x^3}{(x^3-1)^2}$

$= \frac{-3-6x^3}{(x^3-1)^2}$ ✓

$= \frac{-3(1+2x^3)}{(x^3-1)^2}$

2

$(3x)(x^3-1)^{-1}$

$= 3(x^3-1)^{-1} + (3x)(-(x^3-1)^{-2}\cdot 3x^2)$

-3 $9x^3$

(iii) $\frac{d}{dx}(3x-2)^{-4}$

$= -4(3x-2)^{-5} \times 3$

$= \frac{-12}{(3x-2)^5}$ 2

(b) $x^2 - 8x + 5 = 0$

1 (i) $\alpha+\beta = 8$, $\alpha\beta = 5$

2 (ii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 64 - 10$

$= 54$

2 (iii) $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$= 8\,[54-5]$

$= 8 \times 49$

$= 392$

9283 9614

(14)

(c) $A(x-1)^2 + Bx + C \equiv x^2$

put $x = 1$ 2

$B + C = 1$ —— (i)

put $x = 0$

$A + C = 0$

$C = -A$ —— (2)

coeff of $x^2 = A = 1$

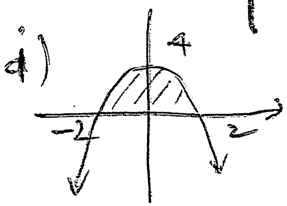
$\therefore C = -1$

$\therefore B = 2$

(d)

1

(d) (i)



(ii)

$-2 \le x \le 2$ 1

Question 3

a) i $12y = x^2 - 4x - 32$

$x^2 - 4x + 4 = 12y + 32 + 4$

$(x-2)^2 = 12y + 36$

$(x-2)^2 = 12(y+3)$

$h = 2$
$a = 3$
$k = -3$

$\boxed{(x-2)^2 = 4 \cdot 3(y+3)}$

ii focus $(h, k+a)$

$= (2, -3+3)$

$= \boxed{(2, 0)}$

vertex (h, k)

$= \boxed{(2, -3)}$

iii Axis of symmetry $\boxed{x = 2}$

iv x intercepts

$x^2 - 4x - 32 = 0$

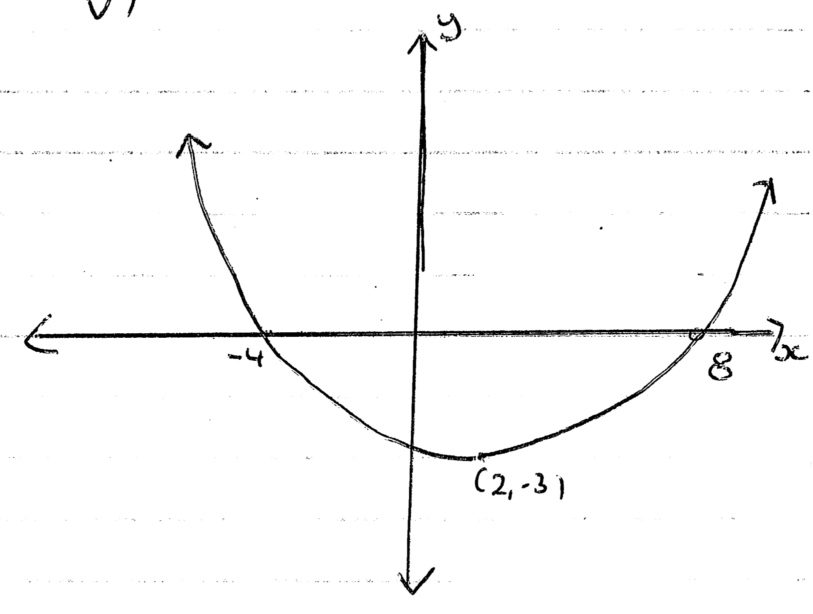
$(x-8)(x+4) = 0$

$\therefore x = 8, -4$

pts of intersection

$\boxed{(8,0) \text{ \& } (-4,0)}$

v)



b) $T_3 = 20$

$20 = ar^2 \Rightarrow a = \frac{20}{r^2}$ ①

$15 = a + ar$ ②

sub ① into ②

$15 = \frac{20}{r^2} + \frac{20\cancel{r}}{r^{\cancel{2}}}$

$15r^2 = 20 + 20r$

$3r^2 - 4r - \overset{-12}{4} = 0$

$\frac{(3r-6)(3r+2)}{3} = 0$

$(r-2)(3r+2) = 0$

$\therefore$ $\boxed{r = 2}$ $3r = -2$

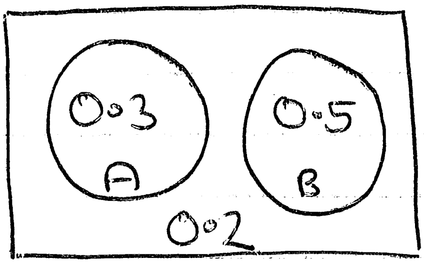
$r = -2/3 \rightarrow |r| > 1$ $\therefore$ $r \neq -2/3$

sub r = 2 into ①

$20 = a \times 2^2$

$\boxed{a = 5}$

c) A & B are mutually exclusive



$P(A \cup B) = 0.3 + 0.5$
$= 0.8$

$P(A \cap B) = 0$

2007 2U ASSESSMENT TASK 1.
SECTION 4.

a) $\log_2(x+4) - \log_2(x-2) = 1.$

$\log_2\left(\frac{x+4}{x-2}\right) = \log_2 2$

$\frac{x+4}{x-2} = 2.$

$x+4 = 2x-4.$

$\underline{\underline{x=8}}$

b) $4^x = 5^{2x-1}.$

$\log_4 4^x = \log_4 5^{2x-1}.$

$x \log_4 4 = 2x-1 \log_4 5.$

$\frac{x}{2x-1} = \log_4 5.$

$x = 2x-1\left(\frac{\log_{10} 5}{\log_{10} 4}\right)$

$+x = 2x-1\ (1.16096)$

$x = 2x(1.169096) - (1.16096)$

$1.3219x = 1.169096$

$x = .0.878.$

$\underline{\underline{x = 0.88\ (2\,dp)}}$

c) $f'(x) = (x+1)(x-3).$

Stationary points occur when $f'(x) = 0$

$\therefore f'(x) = (x+1)(x-3) = 0.$

$x = -1, +3$

c) cont).

Points are given by

$y = \int x^2 - 2x - 3.\,dx.$

$y = \frac{x^3}{3} - x^2 - 3x + C.$

which passes thro $(0,3)$.

$y = 0 - 0 - 0 + C.$

$\underline{\underline{3 = C}}$.

when $x=-1$. $y = -\frac{1}{3} - 1 + 3 + 3$
$= 4\frac{2}{3}.$

A stationary point occurs at $(-1, 4\frac{2}{3})$.

when $x=3$ $y = \frac{27}{3} - 9 + 9 + c.$
$y = -6$

Another stationary point occurs at $(3,-6)$.

<u>NATURE</u>

$\frac{d^2y}{dx^2} = 2x - 2.$

when $x = -1.$
$\frac{d^2y}{dx^2} = -4 < 0$

$\underline{\underline{\therefore \text{maximum at } (-1, 4\tfrac{2}{3})}}$

when $x = 3$
$\frac{d^2y}{dx^2} = 4 > 0.$

$\underline{\underline{\therefore \text{minimum at } (3,-6)}}.$

d) Equation of tangent at (2, −2) where.

$y = x^3 - x^2 - 6$

$m_T = \frac{dy}{dx} = 3x^2 - 2x$ when $x = 2$.

$\frac{dy}{dx} = 3(4) - 4 = 8.$

Point gradient formula.

$y - y_1 = m(x - x_1)$

$y - {}^-2 = 8(x - 2)$

$y = 8x - 18$

This crosses x axis at $y = 0$

$0 = 8x - 18.$

$8x = 18$

$x = 2.25.$

Equation of normal.

$m_N = -\frac{1}{m_T} = -\frac{1}{8}.$

Point gradient (2, −2) $m = -\frac{1}{8}$.

$y - {}^-2 = -\frac{1}{8}(x - 2)$

$y = -\frac{x}{8} - \frac{7}{4}.$

when $y = 0$. $\frac{x}{8} = -\frac{7}{4}.$

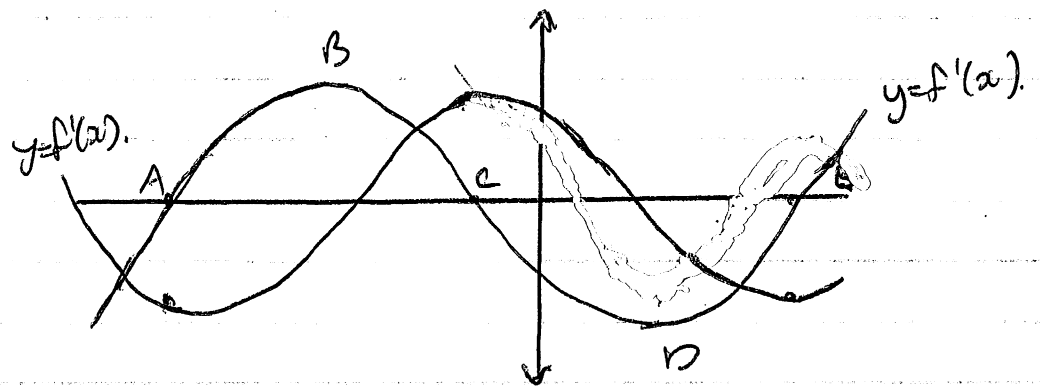
$x = -14$

∴ Distance between T (2.25, 0) and N (14, 0) is 16.25 units

MATH Task 1.

QUESTION 5

a i)



ii) y=f(x) has stationary (turning) points at A, C and E.

iii) The curve y=f(x) has points of inflection at B and D.

b i) $A_1 = 60000\left(1+\frac{r}{1200}\right) - M$

$= 60000(1.0075) - M.$

ii) $A_2 = A_1(1.0075) - M$

$= 60000(1.0075)^2 - (1.0075)M - M.$

iii) $A_1 = PR - M$

$A_2 = PR^2 - MR - M.$

$A_3 = PR^3 - MR^2 - MR - M.$

$A_n = PR^n - MR^{n-1} - MR^{n-2} - \dots - MR - M.$

$= PR^n - M(R^{n-1} + R^{n-2} + \dots + R + 1).$

$= PR^n - M\left(\frac{1-R^n}{1-R}\right).$

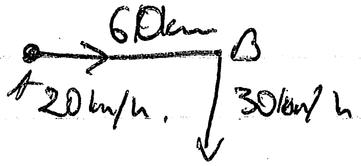
when $A_{120} = 0$,

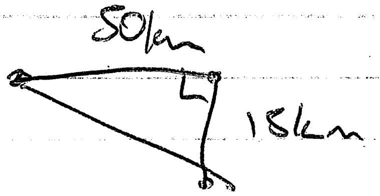
$$0 = 60\,000(1.0075)^{120} - M\left(\frac{1-1.0075^{120}}{1-1.0075}\right)$$

$$M = 60\,000(1.0075)^{120} \times \frac{-0.0075}{1-1.0075^{120}}$$

$$= \$760.05.$$

c(i)





$$AB^2 = 50^2 + 12^2$$
$$= 2725$$

$AB \approx 52$km.

(ii) $AB^2 = (60-20x)^2 + (30x)^2$

$$= 3600 - 2400x + 400x^2 + 900x^2$$

$$AB = 10\sqrt{13x^2 - 24x + 36}$$

(iii)

$$\frac{dAB}{dx} = 5\left(13x^2 - 24x + 36\right)^{-\frac{1}{2}}(26x - 24).$$

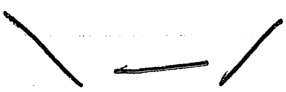
$$= \frac{5(26x-24)}{\sqrt{13x^2 - 24x + 36}}$$

Stat pt $\frac{dAB}{dx} = 0$. $0 = \frac{x}{y}$ when $x = 0$.

$$26x - 24 = 0.$$
$$x = \frac{12}{13}.$$

x	0.	$\frac{12}{13}$	1
$\frac{dAB}{dx}$	-20	0.	2.

$\frac{10}{\sqrt{25}}$



$\therefore x = \frac{12}{13}$ is a minimum

$x = \frac{12}{13}$ hours.